Light energy

Light energy

The ability to carry energy is the main property of light. Let's see how energy of light appears from the Maxwell's equations.

Volumetric energy density of e/m field:

$$w = \underbrace{\frac{\varepsilon_0 E^2}{2}}_{\text{electrical}} + \underbrace{\frac{B^2}{2\mu_0}}_{\text{magnetic}}$$

$$w = \frac{\varepsilon_0 E^2}{2} + \frac{\varepsilon_0 c^2 B^2}{2}$$

Let's take Maxwell's equations for the $\rho = 0$ and $\bar{j} = 0$:

$$\begin{cases} c^2 \operatorname{rot} \bar{B} - \dfrac{\partial \bar{E}}{\partial t} = 0 & (1) \\ \operatorname{rot} \bar{E} + \dfrac{\partial \bar{B}}{\partial t} = 0 & (2) \end{cases}$$

Let's multiply (1) by $\bar{E}$ and (2) by $\bar{B}$:

$$\begin{cases} c^2 \bar{E} \operatorname{rot} \bar{B} = \bar{E} \dfrac{\partial \bar{E}}{\partial t} & (1) \\ c^2 \bar{B} \operatorname{rot} \bar{E} = - c^2 \bar{B} \dfrac{\partial \bar{B}}{\partial t} & (2) \end{cases}$$

Now let's subtract $(1) - (2)$:

$$c^2\left(\bar E[\bar\nabla \bar B] - \bar B[\bar\nabla \bar E]\right) = \bar E\frac{\partial \bar E}{\partial t} + c^2 \bar B\frac{\partial \bar B}{\partial t}$$

However, the right part of this equation is the change of energy density:

$$\frac{\partial w}{\partial t} = \varepsilon_0 \bar E \frac{\partial \bar E}{\partial t} + \varepsilon_0 c^2 \bar B \frac{\partial \bar B}{\partial t}$$

Thus:

$$\varepsilon_0 c^2\left(\bar E[\bar\nabla \bar B] - \bar B[\bar\nabla \bar E]\right) = \frac{\partial w}{\partial t}$$

Little mathematical walkaround

$$\bar\nabla[\bar B\bar E] = \bar E[\bar\nabla \bar B] - \bar B[\bar\nabla \bar E]$$

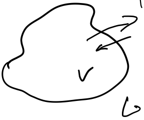
$$\varepsilon_0 c^2\, \bar\nabla[\bar B\bar E] = \frac{\partial w}{\partial t}$$

$$\bar S = \varepsilon_0 c^2[\bar E\bar B]$$ — Umov-Poynting vector

U-P vector is basically the vector of energy density flux.

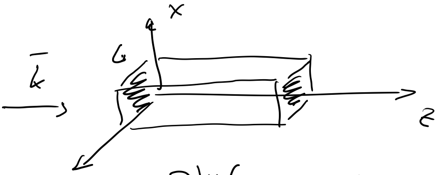
Hence the equation is the energy conservation law:

$$\boxed{\frac{\partial w}{\partial t} = -\operatorname{div}\bar{S}}$$



$$\frac{\partial}{\partial t}\int_V w\,dV = \oint_\sigma \bar{S}\,d\bar{\sigma}$$

We can even make it simple taking rectangular volume:



$$\frac{\partial W}{\partial t} = -\sigma\left(S(t,z) - S(t,0)\right)$$

change of total energy

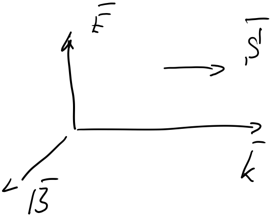
What does it mean?

If the volume does not have charges and currents, then the speed of change of e/m field is solely determined by energy density flux through the surface.

What does it mean?

It means, that in running e/m wave the is directional movement of energy:

$$\bar{S} = \varepsilon_0 c^2 \left[\bar{E}\bar{B}\right]$$



To remind you from "Transverse nature of light":

$$\begin{cases} c^2\left[\bar{k}\bar{B}\right] = -\omega \bar{E} \\ \left[\bar{k}\bar{E}\right] = \omega \bar{B} \end{cases} \quad \text{and} \quad k = \frac{\omega}{c}$$

Thus, taking into account ortogonal nature of vectors $\bar{E}$ and $\bar{B}$:

$$\frac{\omega}{c}|\bar{E}| = \omega B \Rightarrow \boxed{E = cB}$$

$$S = c\varepsilon_0 E^2 = c^3 \varepsilon_0 B^2$$

What does this mean?

It means that volumetric energy density of electric field is equal to that of magnetic field in every point in space and time.

$$S = c(w_e + w_m) = cw$$

Now let's solve this using the wave equation:

$$E(\bar{r}, t) = E_0 \cos(\omega t - \bar{k}\bar{r})$$

Little mathematical walkaround

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

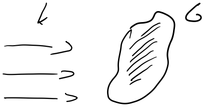
$$S = \frac{c\varepsilon_0 E_0^2}{2}\left(1 + \cos 2(\omega t - \bar{k}\bar{r})\right)$$

This number is giving instantaneous value for energy flux, which is not simple to measure. It is easier to average it:

$$\langle S \rangle = \frac{c\varepsilon_0 E_0^2}{2} \text{ — intensity of light } \underline{I}$$

Light pulse

Since we were able to define the term intensity, let's compare intensity of pulsed and cw radiation.



$I = \frac{P}{\sigma}$ P ~ power

1) Sun constant

$I = 0.15 \ \frac{W}{cm^2}$ —> at all spectrum

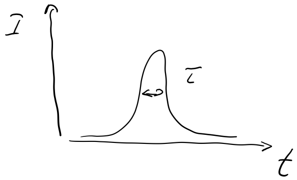
2) He-Ne laser

$P = 10 \ mW$

$w = 1 \ mm$

$I = 0.1 \ \frac{W}{cm^2}$ —> at one particular frequency

3) fs pulse



$\tau = 100 \cdot 10^{-15} \ s$

$P = 1 \ mW$

$R = 1000 \ Hz$

$w = 1 \ mm$

$\Rightarrow I \sim 10^{9} \ \frac{W}{cm^2}$

Let's estimate the field of proton the electron is in:

$e = 1.6 \cdot 10^{-19} \ C$

$r_0 = 5.3 \cdot 10^{-11}\, m$ → first Bohr radius

$\varepsilon_0 = 9 \cdot 10^{-12} \frac{S}{V \cdot m}$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e}{r_0^2} \sim 10^{12} \frac{V}{m}$$

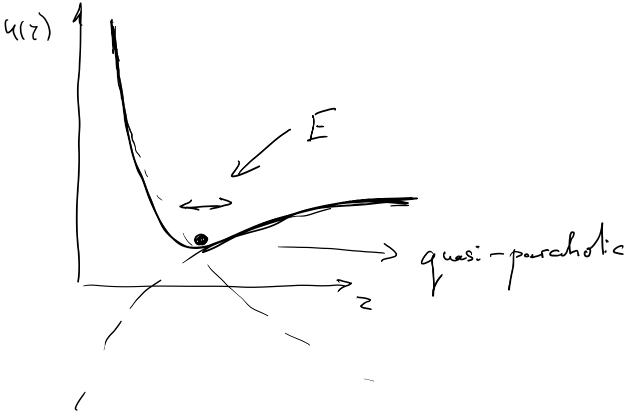
Now let's see what are the fields we can induce with He-Ne and fs:

① He-Ne

$$I = 0.1 \frac{W}{cm^2}$$

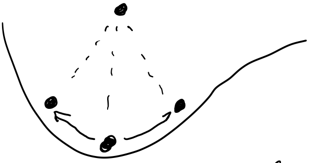
$$E = \sqrt{\frac{2I}{\varepsilon_0 c}} \sim 3 \frac{V}{m}$$

What does it mean?



For He-Ne laser $E_{He-Ne} \ll E_{atom}$

That is small perturbation and described by simple perturbation theory.



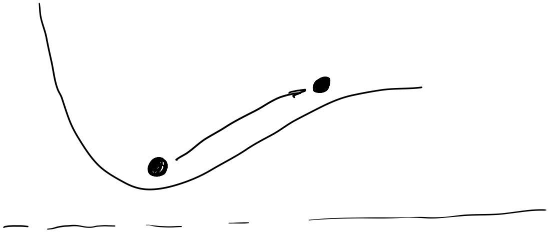
Electron behaves like ball on a spring.

Here we can define linear optics — light matter interaction, when displacement of electron from equilibrated state is minimal — pendulum.

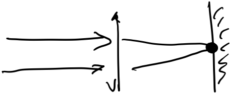
② Let's take fs pulse.

$$E=\sqrt{\frac{2I}{\varepsilon_0 c}} \sim 10^{9}\ \frac{V}{m}$$

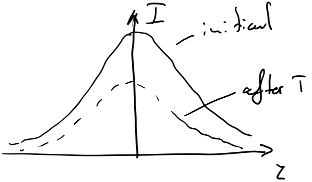
That becomes very comparable to the field of proton. We can kick electron away from the orbit or ionize:



Energy of pulse and heating



Heat diffusion is very slow process. Thus:



$$cm\frac{dT}{dt} = (1-R)\,P(t)$$

R- reflection coefficient

$$\rho = 10^{4}\ \frac{kg}{m^{3}}$$

$$V = 10^{-10}\ cm^{3} = 10^{-16}\ m^{3}$$

$$c = 10^{3}\ \frac{J}{m\cdot K}$$

$$T_{max} = \frac{(1-R)\cdot\int P(t)\,dt}{cm} = \frac{(1-R)\cdot W}{cm} \sim 10^{8} K.$$